Response of Elastic, Piezoelectric and Flexoelectric Beams Through Enhanced Elasticity Theories

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Abstract

The dimensions of many mechanical and electromechanical systems and structures are comparable to those of their material microstructure. In these cases, the microstructure affects the mechanical and electromechanical behavior, witch the classical theories of elasticity and electroelasticity fail to define correctly. For these reason, Gradient Elasticity Theory and Gradient Electroelasticity Theory have been developed. In this work, models of Bernoulli-Euler gradient elastic and gradient electroelastic beams were analyzed. The response to 1D bending, with respect to the deflection, axial strain and electric field were examined.

Expressions od Elastic Energy Density and Electroelastic Enthalpy Density

The nature of Gradient Elasticity Theory and Gradient Electroelasticity Theory, can be understood, observing the expressions of elastic energy density and electroelastic enthalpy density, respectively. In these expressions strain gradient and electric field gradient are introduced.

Classical Elasticity

$$\bar{U} = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}$$

Gradient Elasticity

$$\bar{U} = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + \frac{1}{2} A_{ijklmn} \varepsilon_{ij,k} \varepsilon_{lm,n}$$

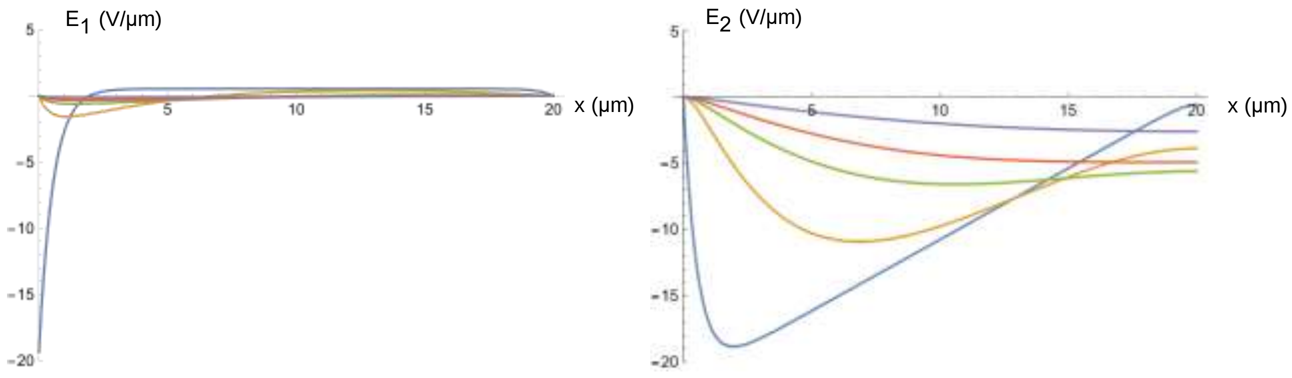
Classical Electroelasticity

$$\bar{H} = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} - \frac{1}{2} \epsilon_{ij} E_i E_j - e_{kij} \varepsilon_{ij} E_k$$

Gradient Electroelasticity

$$\bar{H} = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + \frac{1}{2} A_{ijklmn} \varepsilon_{ij,k} \varepsilon_{lm,n} - \frac{1}{2} \epsilon_{ij} E_i E_j - \frac{1}{2} \alpha_{ijk} E_{i,j} E_{k,l} - \frac{1}{2} e_{kij} \varepsilon_{ij} E_k$$

Response of PZT-7A Gradient Electroelastic Beam



The response of a cantilever beam, constructed by ceramic material PZT-7A. The length of the beam is 20 μm. The transverse cross-section is square of 1 μm width. On the free end, a concentrated load of 0.1 mN is enforced. At the left diagram, the axial component of electric field is presented and at the right diagram the transverse component of electric field is presented, for g = 0, g = 0.1μm, g = 0.2 μm, g = 0.3 μm and g = 0.5 μm (blue, yellow, green, red and purple line, respectively). Where g, stands for the microstructure characteristic length, witch is a measure of microstructure influence.

Conclusions

Gradient elastic and gradient electroelastic beams present lower response, compered with classical elastic and classical electroelastic ones. As the microstructure characteristic length is increased, the response is decreased. This means that as the dimensions of the beam become smaller (and therefore closer to those of the microstructure), the microstructure effect increases, giving greater rigidity